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NONLINEAR ANALYSIS OF PLASMA INSTABILITY EXCITED BY THE AURORAL ELECTROJET

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Nonlinear analysis of the Farley-Buneman instability of the auroral electrojet used to derive the magnitude of ionospheric irregularities and their direction of propagation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The two-fluid (electron and ion) continuity and momentum equations are expanded in terms of the amplitudes of the fluctuations, and the nonlinear dispersion relation is derived. From this expression the threshold condition (when the growth rate = 0) as well as the condition for instability (when the growth rate < 0) are obtained. In addition, the order of magnitude of the density fluctuations is estimated which agrees with that observed by Booker. It is shown that these results can be obtained only with the nonlinear theory. (Cont'd on reverse)

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20. Finally, it is shown that as a result of electric field measurements by Kelley and Mozer our theory predicts that the wave propagation vector of the irregularities must have a small, but nonvanishing, component parallel to the east-west direction.

1. Introduction

During a magnetospheric substorm the plasma in the E-region of the auroral ionosphere is observed to be unstable (Booker, 1960). The instability is due to a relative drift between ions and electrons in a direction perpendicular to the geomagnetic field lines. Such a drift can occur frequently in the auroral ionosphere at an altitude of approximately 110 km because of the presence of an electric field directed normal to the ambient magnetic field. In this region the motion of the ions is somewhat restricted by collisions with the neutral particles, while at the same time the lighter electrons are more or less free to execute a drifting motion with a velocity equal to $\underline{E} \times \underline{B}/B^2$.

Recently Lee _et al._ (1971) showed that a high density plasma ($N_e \geq 10^5$ cm^{-3}) is susceptible to a high frequency, short wavelength type instability, if there exists a sufficiently large drift velocity between ions and electrons perpendicular to the background magnetic field, called the high-frequency Hall current instability (Lee _et al._, 1971). Their linear analysis of this instability results in a threshold drift velocity $V_D \geq 3(k_B\ T/M)$, where k_B is Boltzmann's constant, $T = T_e \cong T_i$ is the plasma temperature, and M is the ion mass. However, observations by Kelley and Mozer (1973) seem to indicate that nonlinear effects somehow stabilize the waves at a speed approximately equal to the ion acoustic velocity and less than the electron drift velocity.

Following the work of Rogister (1971) on the equatorial electrojet we give a nonlinear analysis of the high frequency Hall current two-stream instability applicable to the auroral electrojet. Our results show that the unstable waves indeed travel at a speed less than the electron drift velocity. Moreover we made an order of magnitude estimate of the density fluctuation, and this agrees well with that observed by Booker (1960). Also the proper direction of the propagation of the unstable waves is shown.

2. Discussion of the Macroscopic Equations

In the region of the auroral electrojet electron-ion and ion-ion collisions are negligible compared with collisions between neutrals and charged particles. Moreover ionization and recombination processes may be ignored. We shall use the two-fluid equations to describe the simple model of the electrojet layer. In this framework the fundamental equations are

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \underline{V}_j) = 0 \tag{1}$$

$$nm_j \left(\frac{\partial \underline{V}_j}{\partial t} + \underline{V}_j \cdot \nabla \underline{V}_j\right) = -\nabla p_j + nq_j \left(\underline{E} + \frac{1}{c} \underline{V}_j \times \underline{B}\right) - \nu_j nm_j \underline{V}_j \tag{2}$$

where $j = e, i$ refers to the electrons and ions respectively, and ν_j is the collision frequency between the j^{th} charged particle and a neutral particle. In equations (1) and (2) charge separation and finite Larmor radius effects are not included; these may be shown to be negligible in the region of the auroral electrojet.

To start the analysis we decompose each variable ϕ according to

$$\phi = \tilde{\phi} + \delta\phi \tag{3}$$

where $\tilde{\phi}$ is the spatial average of ϕ, and $\delta\phi$ is a fluctuation component, such that its average $\langle \delta\phi \rangle = 0$. In accordance with the Farley-Buneman type instability we consider electrostatic waves with propagation vector perpendicular to the ambient magnetic field. Also we shall only consider fluctuations with very small parallel wave numbers as these are more susceptible to resistive instabilities (Rosenbluth, 1965). Furthermore, for convenience we shall assume one-dimensional propagation, i.e. we let $\nabla \equiv \hat{s}(\partial/\partial s)$, where $\hat{s}$ is a unit vector along the direction of wave propagation.

We now assume that the average quantities are in equilibrium, and expand equation (1) as follows:

$$\frac{\partial \tilde{n}}{\partial t} = 0 \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \tilde{\underline{V}}_j \cdot \hat{s}\,\frac{\partial}{\partial s}\right)\delta n + \tilde{n}\,\frac{\partial}{\partial s}\,(\hat{s}\cdot\delta\underline{V}_j) + \frac{\partial}{\partial s}(\hat{s}\cdot\delta\underline{V}_j\,\delta n) = 0 \tag{5}$$

Similarly, expanding eq. (2) one obtains

$$\frac{\partial \tilde{\underline{V}}_j}{\partial t} + \frac{\partial}{\partial t}(\delta\underline{V}_j) + (\tilde{\underline{V}}_j + \delta\underline{V}_j)\cdot\hat{s}\,\frac{\partial}{\partial s}\,(\delta\underline{V}_j) + \nu_j(\tilde{\underline{V}}_j + \delta\underline{V}_j) = \frac{-\hat{s}\,\frac{\partial}{\partial s}(\delta p_j)}{(\tilde{n} + \delta n)m_j}$$

$$+ \frac{q_j}{m_j}\,[\underline{E} + \frac{1}{c}(\tilde{\underline{V}}_j + \delta\underline{V}_j) + \underline{B}] \tag{2a}$$

where use has been made of the relations

$$\frac{\partial}{\partial s}\,(\tilde{p}_j) = \frac{\partial}{\partial s}\,(\tilde{n}) = \frac{\partial}{\partial s}(\tilde{\underline{V}}_j) = 0$$

Taking the average of eq. (2a) and noting that

$$\langle\delta\underline{V}_j\rangle = 0 \text{ and } \langle(\tilde{\underline{V}}_j\,\cdot\,\hat{s})\frac{\partial}{\partial s}\,\delta\underline{V}_j\rangle = (\tilde{\underline{V}}_j\,\cdot\,\hat{s})\frac{\partial}{\partial s}\,\langle\delta\underline{V}_j\rangle = 0$$

one obtains

$$\frac{\partial \tilde{\underline{V}}_j}{\partial t} + \langle(\hat{s}\,\cdot\,\delta\underline{V}_j)\,\frac{\partial}{\partial s}\,\delta\underline{V}_j\rangle = \langle-\,\frac{\hat{s}\,\frac{\partial}{\partial s}(\delta p_j)}{(\tilde{n} + \delta n)m_j}\rangle$$

$$+ \frac{q_j}{m_j}\,[\underline{E} + \frac{1}{c}\,\tilde{\underline{V}}_j\,\times\,\underline{B}] - \nu_j\,\tilde{\underline{V}}_j \tag{2b}$$

Subtracting eq. (2b) from eq. (2a) one obtains

$$\frac{\partial \delta \underline{V}_j}{\partial t} + (\tilde{\underline{V}}_j + \delta \underline{V}_j) \cdot \hat{s} \frac{\partial}{\partial s} \delta \underline{V}_j - <\hat{s} \cdot \delta \underline{V}_j \frac{\partial}{\partial s} \delta \underline{V}_j> =$$

$$\frac{-\hat{s}\frac{\partial}{\partial s} \delta p_j}{(\tilde{n} + \delta n)\, m_j} + <\frac{\hat{s} \frac{\partial}{\partial s} \delta p_j}{(\tilde{n} + \delta n) m_j}> + \frac{q_j}{m_j} \frac{1}{c} (\delta \underline{V}_j \times \underline{B}) - \nu_j \tilde{\underline{V}}_j \tag{6}$$

Using Cartesian coordinates, we let $\underline{B} = B(-\hat{i}_z)$ and $\hat{s} = s_x \hat{i}_x + s_y \hat{i}_y$ such that $\hat{s} \cdot \hat{i}_z = 0$. Taking the vector product of $\hat{s}$ with eq. (6) and followed by the scalar product with $\hat{i}_z$, we obtain

$$\Omega_j^{-1} \hat{i}_z \cdot \hat{s} \times \{\frac{\partial \delta \underline{V}_j}{\partial t} + (\underline{V}_j + \delta \underline{V}_j) \cdot \hat{s} \frac{\partial}{\partial s} \delta \underline{V}_j - <\delta \underline{V}_j \cdot \hat{s} \frac{\partial}{\partial s} \delta \underline{V}_j>$$

$$+ \nu_j \delta \underline{V}_j\} = \hat{s} \cdot \delta \underline{V}_j \tag{7}$$

where $\Omega_j \equiv \frac{q_j B}{m_j c}$ is the Larmor frequency of the j^{th} particle. Also, taking the scalar product of $\hat{s}$ with eq. (6), and summing over all electrons and ions, we obtain the following equation

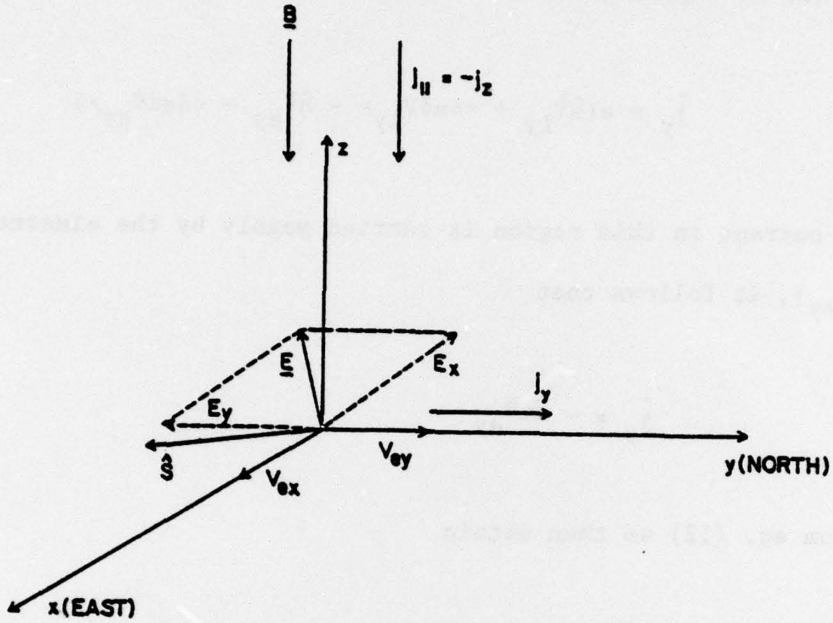
$$\sum_{j=e,i} m_j \hat{s} \cdot \{\frac{\partial}{\partial t} \delta \underline{V}_j + (\underline{V}_j + \delta \underline{V}_j) \cdot \hat{s} \frac{\partial}{\partial s} \delta \underline{V}_j + \nu_j \delta \underline{V}_j\} =$$

$$- \sum_{j=e,i} \frac{\frac{\partial}{\partial s} \delta p_j}{(\tilde{n} + \delta n)} + \sum_{j=e,i} <\frac{\frac{\partial}{\partial s} \delta p_j}{(\tilde{n} + \delta n)}> + \sum_{j=e,i} q_j \underline{B} \cdot (\hat{s} \times \frac{\delta \underline{V}_j}{c}) \tag{8}$$

where we have used the simplification

$$\hat{s} \cdot <\delta \underline{V}_j \cdot \hat{s} \frac{\partial}{\partial s} \delta \underline{V}_j> = \frac{1}{2} <\frac{\partial}{\partial s} (\hat{s} \cdot \delta \underline{V}_j)^2> = 0$$

From the Birkeland current system we note that the field-aligned current feeds into the south edge of the auroral arc (see Fig. 1).



From the current continuity equation we then have

$$\nabla \cdot \underline{j} = 0 \tag{9}$$

or

$$\frac{\partial j_y}{\partial y} = \frac{\partial j_{||}}{\partial z} \tag{10}$$

j_y is the current density in the South-North direction, and $j_{||}$ is the Birkeland current which feeds into the auroral arc. The current in the y-direction is largely carried by the electrons on account of their high mobility.

Now, $$j_y = ne\, V_{iy} - ne\, V_{ey} \tag{11}$$
and using (3) to expand (11), and taking the average, we obtain

$$\tilde{j}_y = e(\tilde{n}\tilde{V}_{iy} + \langle\delta n \delta V_{iy}\rangle - \tilde{n}\tilde{V}_{ey} - \langle\delta n \delta V_{ey}\rangle) \tag{12}$$

Since the current in this region is carried mainly by the electrons ($V_{iy} << V_{ey}$), it follows that

$$\tilde{j}_y \simeq - \tilde{n}e\tilde{V}_{ey}$$

Hence, from eq. (12) we then obtain

$$\langle\delta n \delta V_{ey}\rangle \cong \tilde{n}\, \tilde{V}_{iy} + \langle\delta n \delta V_{iy}\rangle \tag{13}$$

We are now considering the two-dimensional flow across the ambient magnetic field. With the magnetic field $B\hat{i}_z$ and the primary electric field E_x in the East-West direction given we have a system of six equations in the seven average fluid variables $\tilde{\underline{V}}_j$, $\tilde{n}$, E_y, and p. Hence, we also need the equation of state $p = nk_BT$ to complete the description of the system. Here it has been assumed that at the altitude region around 110 km electrons and ions are in thermal equilibrium, i.e. $T_e \simeq T_i = T$, and T is considered to be known.

3. Derivation of the Nonlinear Development of the Instability

From observations the mean square density fluctuation, after stabilization is estimated to be of the order of $\frac{\langle\delta n \delta n\rangle}{\tilde{n}^2} \sim 3 \times 10^{-3}$ (Booker, 1960). Correspondingly we consider a small quantity ε, say $10^{-2}<\varepsilon<10^{-1}$, such that we may write

$$\delta n \sim \epsilon \tilde{n} \tag{14}$$

Also the experimental observations of Kelley and Mozer (1973) show that $|E_x| \sim |E_y|$. This means that the primary electric field is of the same order as the secondary electric field. This implies that the auroral electrojet has a current in the East-West direction which is about the same order as that in the y-direction. It is to be noted that the current in the y-direction results from the continuity equation involving the field-aligned current. Thus we may conclude that the electron drift velocity $|V_{ex}| \sim |V_{ey}|$. Based on observations in the auroral ionosphere we now introduce the following order of magnitude approximations:

$$\begin{aligned}
&\nu_e \sim \epsilon\, \Omega_e \\
&\nu_i \sim \epsilon\, \nu_e \sim \epsilon^2 \Omega_e \\
&\Omega_i \sim \epsilon \nu_i \sim \epsilon^3 \Omega_e \\
&\omega \sim \nu_i \sim \epsilon^2 \Omega_e \\
&c_s \sim \tilde{V}_e \\
&\lambda = \frac{2\pi c_s}{\omega}
\end{aligned} \tag{15}$$

where $c_s = \{k_B(T_e + T_i)/m_i\}^{1/2}$ = ion sound speed.

and

$$\tilde{V}_i \sim \left(\frac{k_B T}{m_i}\right)^{1/2} \sim \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{k_B T}{m_e}\right)^{1/2} \sim \epsilon \tilde{V}_e \tag{16}$$

where $T = T_e \simeq T_i$ is the plasma temperature, and $\frac{m_e}{m_i} \sim 10^{-4}$ in order of magnitude. Using the order of magnitude approximations as indicated in (15) we find from equation (13) that

$$\delta V_{ey} \sim \tilde{V}_{ey}$$

and

$$\delta V_{iy} \sim \tilde{V}_{iy} \sim \varepsilon \tilde{V}_e$$

We now expand the governing equation (4)-(8), and (13) in powers of ε, using the relations (15,-(18). To lowest order, it is found that the waves propagate without changing form. This means that the growth rates of instabilities and also the dispersive effects are small compared with the real part of the frequencies and proceed on larger time scales. To proceed to the next order in ε we use the multiple time scales expansion scheme of Bogoliubov and Krylov (1947), i.e. let

$$\delta n(t) \equiv n\,(t_o;\ t_1;\ t_2;\ldots)$$

where $t_1 = \varepsilon t_o$, $t_2 = \varepsilon t_1$,... etc.

Also let $\delta n = \delta n^{(o)} + \varepsilon \delta n^{(1)} + \ldots$

Then,

$$\frac{\partial}{\partial t}\,\delta n(t) \equiv \frac{\partial}{\partial t}\,\delta n^{(o)} + \varepsilon\,\Big(\frac{\partial \delta n^{(o)}}{\partial t_1} + \frac{\partial \delta n^{(1)}}{\partial t_o}\Big) + \ldots$$

From now on the time variable will be expressed in terms of t_o, and the subscript $_o$ will be dropped although it is always implied.

We expand equation (7) by means of the scaling relations (13) and compare the order of magnitude of each individual term:

$$(\varepsilon^2,\ \varepsilon^2,\ \varepsilon^2,\ \varepsilon^2,\ \varepsilon,\ 1)$$

Hence, we find that

$$\hat{s} \cdot \delta \underline{V}_e^{(o)} = 0$$

$$\hat{s} \cdot \delta \underline{V}_e^{(1)} = \Omega_e^{-1} \nu_e \hat{i}_z \cdot (\hat{s} \times \delta \underline{V}_e^{(o)}) \tag{23}$$

and

$$\Omega_e^{-1} \hat{i}_z \cdot \hat{s} \times \{\frac{\partial}{\partial t} \delta \underline{V}_e^{(o)} + (\tilde{\underline{V}}_e \cdot \hat{s}) \frac{\partial}{\partial s} \delta \underline{V}_e^{(o)} + \nu_e \delta \underline{V}_e^{(1)}\} = \hat{s} \cdot \delta \underline{V}_e^{(2)} \tag{24}$$

From eq. (5) with j = e the orders of magnitude of the individual terms are

$$(1,\ 1,\ \varepsilon^{-1},\ 1)$$

and eq. (5) splits into

$$\frac{\partial}{\partial s} (\hat{s} \cdot \delta \underline{V}_e^{(o)}) = 0 \tag{25}$$

$$[\frac{\partial}{\partial t} + (\tilde{\underline{V}}_e \cdot \hat{s}) \frac{\partial}{\partial s}] \delta n^{(o)} + \tilde{n} \frac{\partial}{\partial s} (\hat{s} \cdot \delta \underline{V}_e^{(1)}) + \frac{\partial}{\partial s} [(\hat{s} \cdot \delta \underline{V}_e^{(o)}) \delta n^{(o)}] = 0 \tag{26}$$

and

$$(\frac{\partial}{\partial t} + \tilde{\underline{V}}_e \cdot \hat{s}) \delta n^{(1)} + \varepsilon \frac{\partial}{\partial t} \delta n^{(o)} + \tilde{n} \frac{\partial}{\partial s} (\hat{s} \cdot \delta \underline{V}_e^{(2)}) + \frac{\partial}{\partial s} [(\hat{s} \cdot \delta \underline{V}_e^{(1)}) \delta n^{(o)}]$$
$$+ \frac{\partial}{\partial s} [(\hat{s} \cdot \delta \underline{V}_e^{(o)}) \delta n^{(1)}] = 0 \tag{27}$$

On the other hand the orders of magnitude of the individual terms of eq. (8) with j = i(e) are

$$[1(\varepsilon^2),\ \varepsilon(\varepsilon^2),\ \varepsilon(\varepsilon^2),\ 1(\varepsilon),\ 1(1),\ 1(1),\ \varepsilon(1)]$$

where the orders of magnitude of the electron terms are given in parentheses. Hence, eq. (8) becomes

$$\hat{s} \cdot \{\frac{\partial \delta \underline{V}_i^{(o)}}{\partial t} + \nu_i \delta \underline{V}_i^{(o)}\} = - \frac{c_s^2}{\tilde{n}} \frac{\partial \delta n^{(o)}}{\partial s} + \Omega_i \hat{i}_z \cdot \hat{s} \times \delta \underline{V}_e^{(o)} \tag{28}$$

and

$$\hat{s} \cdot \{\frac{\partial \delta \underline{V}_i^{(1)}}{\partial t} + \varepsilon \frac{\partial \delta \underline{V}_i^{(o)}}{\partial t} + (\underline{V}_i + \delta \underline{V}_i^{(o)}) \cdot \hat{s} \frac{\partial}{\partial s} \delta \underline{V}_i^{(o)} + \nu_i \delta \underline{V}_i^{(1)}\}$$

$$+ \frac{m_e}{m_i} \nu_e \hat{s} \cdot \delta \underline{V}_e^{(o)} = - \frac{c_s^2}{\tilde{n}} \frac{\partial \delta n^{(1)}}{\partial t} - \varepsilon \frac{s^2}{\tilde{n}} \frac{\partial \delta n^{(o)}}{\partial t} - \Omega_i \hat{i}_z \cdot \hat{s} \times \delta \underline{V}_i^{(o)}$$

$$+ \Omega_e \hat{i}_z \cdot \hat{s} \times \delta \underline{V}_e^{(1)} \tag{29}$$

Note that the term $\left\langle \frac{c_s^2 \frac{\partial \delta n^{(1)}}{\partial s}}{\tilde{n}} \right\rangle$ disappears as

$$\left\langle \frac{c_s^2 \frac{\partial \delta n^{(1)}}{\partial s}}{\tilde{n}} \right\rangle = \frac{c_s^2}{\tilde{n}} \frac{\partial}{\partial s} \langle \delta n^{(1)} \rangle = 0$$

The orders of magnitude of the individual terms of eq. (5) with j = i are

$$(1, \varepsilon, 1, \varepsilon)$$

Hence, eq. (5) with j = i can be decomposed into

$$\frac{\partial \delta n^{(o)}}{\partial t} + \tilde{n} \frac{\partial}{\partial s} (\hat{s} \cdot \delta \underline{V}_i^{(o)}) = 0 \tag{30}$$

and

$$\frac{\partial \delta n^{(1)}}{\partial t} + \varepsilon \frac{\partial \delta n^{(o)}}{\partial t} + (\underline{V}_i \cdot \hat{s}) \frac{\partial}{\partial s} (\delta n^{(o)}) + \tilde{n} \frac{\partial}{\partial s} (\hat{s} \cdot \delta \underline{V}_i^{(1)})$$

$$+ \frac{\partial}{\partial s} (\hat{s} \cdot \delta \underline{V}_i^{(o)} \delta n^{(o)}) = 0 \tag{31}$$

From eqs. (23), (26), (28), and (30) we obtain

$$(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}) \frac{\partial \delta n^{(o)}}{\partial t} + (\tilde{\underline{V}}_e \cdot \hat{s}) \frac{\partial \delta n^{(o)}}{\partial s} - \frac{\nu_e}{\Omega_e \Omega_i} (\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial s^2}) \delta n^{(o)} = 0 \tag{32}$$

Eq. (32) represents the dispersion relation for a density wave propagating in a direction perpendicular to the ambient magnetic field. We now let the density perturbation be of the form

$$\delta\, n^{(o)} = \delta \underline{n}^{(o)} \exp\left[i\,(\omega t - ks)\right] \tag{33}$$

where $\omega = \omega_r + i\omega_i$ $(\omega_r,\ \omega_i$ real)

First we let $\omega_i = o$, and substitute the form of $\delta n^{(o)}$ as given by (33) into the dispersion relation (32). We obtain the phase velocity of the wave

$$c_s = \frac{\omega_r}{k} = (\tilde{\underline{V}}_e \cdot \hat{s})\left(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right)^{-1} \tag{34}$$

Moreover eq. (32) becomes:

$$\left(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right) \frac{\partial \delta n^{(o)}}{\partial t} + (\tilde{\underline{V}}_e \cdot \hat{s}) \frac{\partial}{\partial s} \delta n^{(o)} = 0 \tag{35}$$

which yields the solution

$$\delta n^{(o)}(s;\ t;\ \varepsilon t;\ \ldots) = \delta \underline{n}^{(o)}\left[s - t(\tilde{\underline{V}}_e \cdot \hat{s})\left(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right)^{-1};\ \varepsilon t;\ \ldots\right] \tag{36}$$

which shows that the profile of the density wave propagates without distortion on the "fast" time scale under the condition given by eq. (34). On the other hand, the condition for instability may be obtained by setting $\omega_i < 0$ in the dispersion relation (32). Thus waves with phase velocities given by

$$c_s < \frac{\omega_r}{k} < (\tilde{\underline{V}}_e \cdot \hat{s})\left(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right)^{-1} \tag{37}$$

will be unstable.

4. Order of Magnitude Estimate of the Fluctuation.

We now substitute (36) into eq. (30) and obtain

$$\frac{\partial}{\partial s}(\hat{s} \cdot \delta \underline{V}_i^{(o)}) = -\frac{1}{\tilde{n}}\frac{\partial n^{(o)}}{\partial t} = \frac{1}{\tilde{n}}(\tilde{\underline{V}}_e \cdot \hat{s})(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \frac{\partial}{\partial s} \delta n^{(o)}$$

i.e.

$$\hat{s} \cdot \delta \underline{V}_i^{(o)} = \frac{1}{\tilde{n}}(\tilde{\underline{V}}_e \cdot \hat{s})(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \delta n^{(o)} \tag{38}$$

This relation is then substituted into eq. (28), and we make use of the threshold relation (34). Also we let

$$\hat{s} = s_x \hat{i}_x + s_y \hat{i}_y \tag{39}$$

$$\delta \underline{V}_e^{(o)} = \delta V_{ex}^{(o)} \hat{i}_x + \delta V_{ey}^{(o)} \hat{i}_y \tag{40}$$

Noting that $\hat{s} \cdot \delta \underline{V}_e^{(o)} = 0$, eq. (28) then becomes

$$\delta V_{ey}^{(o)} = (\hat{s} \cdot \hat{i}_x) \frac{1}{\tilde{n}} \frac{\nu_i}{\Omega_i} (\tilde{\underline{V}}_e \cdot \hat{s})(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \delta n^{(o)} \tag{41}$$

Therefore

$$\langle \delta n^{(o)} \delta V_{ey}^{(o)} \rangle = (\hat{s} \cdot \hat{i}_x) \frac{1}{\tilde{n}} \frac{\nu_i}{\Omega_i} (\tilde{\underline{V}}_e \cdot \hat{s})(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \langle \delta n^{(o)} \delta n^{(o)} \rangle$$

or,

$$\langle \delta n \delta V_{ey} \rangle = (\hat{s} \cdot \hat{i}_x)(\frac{1}{\tilde{n}} \frac{\nu_i}{\Omega_i})(\tilde{\underline{V}}_e \cdot \hat{s})(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \langle \delta n \delta n \rangle \tag{42}$$

The orders of magnitude of the terms in eq. (2b) with $j = i$ are $(1, \varepsilon, \varepsilon^2, 1, \varepsilon, 1)$.

Hence, to zeroth order eq. (2b) becomes

$$\frac{\partial \tilde{\underline{V}}_i}{\partial t} = \frac{q_i}{m_i} \underline{E} - \nu_i \tilde{\underline{V}}_i \tag{43}$$

As assumed previously, the average quantities are in a state of equilibrium, so

$$\frac{\partial \underline{\tilde{V}}_i}{\partial t} = 0 \tag{44}$$

Hence, eq. (43) reduces to

$$\underline{\tilde{V}}_i = \frac{q_i}{m_i \nu_i} \underset{\sim}{E} \quad ,$$

and, in particular, its y-component for the ions becomes

$$\tilde{V}_{iy} = \frac{e}{m_i \nu_i} E_y \tag{45}$$

By a similar analysis eq. (2b) with j = e becomes to lowest order

$$\underline{E} + \frac{1}{c} \underline{\tilde{V}}_e \times \underline{B} = 0 \tag{46}$$

In component form we thus have

$$\begin{aligned} \tilde{V}_{ex} &= - \frac{c}{B} E_y \\ \tilde{V}_{ey} &= \frac{c}{B} E_x \end{aligned} \tag{47}$$

Hence, $\underline{\tilde{V}}_e \cdot \hat{s} = (\tilde{V}_{ex} s_x + \tilde{V}_{ey} \; s_y)$

$$= \frac{c}{B} \left[E_x (\hat{s} \cdot \hat{i}_y) - E_y (\hat{s} \cdot \hat{i}_x) \right] \tag{48}$$

Combining (48), (42), (45), and (13) yields

$$(\hat{s} \cdot \hat{i}_x) \frac{1}{\tilde{n}} \frac{\nu_i}{\Omega_i} \frac{c}{B} [E_x(\hat{s} \cdot \hat{i}_y) - E_y(\hat{s} \cdot \hat{i}_x)] \left[1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right]^{-1} <\delta n \delta n>$$

$$= \tilde{n} \frac{c}{B} \frac{\Omega_i}{\nu_i} E_y \quad , \tag{49}$$

where $<\delta n \delta V_{iy}>$ has been neglected compared with $\tilde{n} \tilde{V}_{iy}$.

Rearranging the above equation, we obtain

$$E_y = \{E_x(\hat{s} \cdot \hat{i}_x)(\hat{s} \cdot \hat{i}_y) \frac{\nu_i^2}{\Omega_i^2} (1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \frac{<\delta n \delta n>}{\tilde{n}\tilde{n}} [1 +$$

$$(\hat{s} \cdot \hat{i}_x) \frac{\nu_i^2}{\Omega_i^2} (1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i})^{-1} \frac{<\delta n \delta n>}{\tilde{n}\tilde{n}}]^{-1} \tag{50}$$

The secondary electric field $E_y = \frac{m_i \nu_i \tilde{V}_{iy}}{e}$ decreases due to the fact that the ion flux $n\tilde{V}_{iy}$ in the y-direction decreases. Hence, according to the first of eq's (47), the East-West electron flux also decreases until the nonlinear density perturbation in the wave reaches a saturated state,

i.e. $$\tilde{V}_{ex} = c_s (1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}) = - \frac{c}{B} E_y, \tag{51}$$

according to eq. (34).

Suppose the wave propagates in the East-West direction. Then,

$$\hat{s} \cdot \hat{i}_x = 1 \quad \text{and} \quad \hat{s} \cdot \hat{i}_y = 0,$$

and, from (50), we see that, if $E_y \neq 0$, we have

$$[1 + \frac{\nu_i^2}{\Omega_i^2}\left[1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right]^{-1} \frac{<\delta n \delta n>}{\bar{n}\bar{n}}] = 0$$

i.e.
$$\frac{<\delta n \delta n>}{\bar{n}\bar{n}} = - \frac{\Omega_i^2}{\nu_i^2} (1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}) \tag{52}$$

It was observed by Whalen and McDiarmid (1972) that the auroral electron precipitation is practically field-aligned as the rocket payload passed through the Northern edge of a visible auroral display. Based on this observation we can determine the magnitude of the electron precipitation by calculating the field-aligned current. The latter is related to the horizontal current by the current continuity equation $\nabla \cdot j = 0$. Thus we have

$$\frac{\partial j_{||}}{\partial z} = \frac{\partial j_y}{\partial y}$$

where

$$j_y = \sigma_p E_y + \sigma_H E_x \tag{53}$$

σ_p and σ_H are the height integrated Pedersen and Hall conductivity, respectively, and $j_{||}$ represents the field-aligned current density.

Substituting eq. (50) into (53), we obtain

$$j_y = \sigma_p E_x (\hat{s} \cdot \hat{i}_x)(\hat{s} \cdot \hat{i}_y) \frac{\nu_i^2}{\Omega_i^2} \left[1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right]^{-1} \frac{<\delta n \delta n>}{\bar{n}\bar{n}} [1 +$$
$$(\hat{s} \cdot \hat{i}_x) \frac{\nu_i^2}{\nu_i^2}\left[1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right]^{-1} \frac{<\delta n \delta n>}{\bar{n}\bar{n}}]^{-1} + \sigma_H E_x$$

If now, for convenience, we assume that this current is uniform over the arc width, then we may determine the field-aligned current density $j_{||}$. Consequently the electron precipitation may be calculated.

5. Conclusions

Based on the Farley-Buneman two-stream instability (Buneman, 1963; Farley, 1963) we have developed a nonlinear analysis of the auroral electrojet. The electron temperature is assumed to be equal to the ion temperature in the electrojet. The results of our analysis indicate the following points of interest.

1) The nonlinear analysis of our two-fluid model is able to predict the order of magnitude of the irregularities of the auroral electrojet. Obviously this is outside the scope of a linear theory. If we use the results of Kelley and Mozer's (1973) observation that the primary electric field E_x is approximately equal to the secondary electric field E_y (in the South-North direction), then

$$\frac{\langle \delta n \delta n \rangle}{\bar{n}\bar{n}} \simeq \frac{\Omega_i^{\,2}}{\nu_i^{\,2}} \left(1 - \frac{\nu_e \nu_i}{\Omega_e \Omega_i}\right) = O(10^{-3})$$

This fluctuation level is of the same order of magnitude as that observed by Booker (1960).

2) The nonlinear theory shows that the electron drift velocity for instability is consistently larger than the phase velocity of the unstable wave. The latter is slightly larger than, or equal to, the ion-acoustic velocity. This result is in contrast to the linear theory of the Farley-Buneman instability, where the drift velocity is shown to be larger than the ion thermal velocity.

3) From eq. (50) we note that when the direction of the wave propagation is exactly in the y-direction, then E_y is reduced to zero. This not only does not agree with Kelley and Mozer's observations which indicates a non-zero E_y field, but also contradict our assumption of the Birkeland current model. Therefore, we conclude that the wave propagation vector should have a small, but non-zero, component in the x-direction.

References

Bogoliubov, N. N. and N. Krylov, Introduction to Nonlinear Mechanics, Princeton Univ. Press, N.J. (1947).

Booker, H. G., Physics of the Upper Atmosphere (1960).

Buneman, O. Phys. Rev. Lett., 10, 285 (1963).

Farley, D. T., Jr., J. Geophys. Res., 68, 6083 (1963).

Kelley, M. C. and F. S. Mozer, J. Geophys. Res., 78, 2214 (1973).

Lee, K., C. F. Kennel and J. M. Kindel, Radio Sci., 6, 209 (1971).

Rogister, A., J. Geophys. Res., 76, 7754 (1971).

Rosenbluth, M. N., Microinstabilities, in Plasma Physics, p. 485. Int. Atomic Energy Agency, Vienna (1965).

Whalen, B. A. and J. B. McDiarmid, J. Geophys. Res., 77, 191 (1972).